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und Luftverkehr

GNSS Under Attack

How GNSS Works – A Technical Primer for All

Lecture – 9:15-9:45

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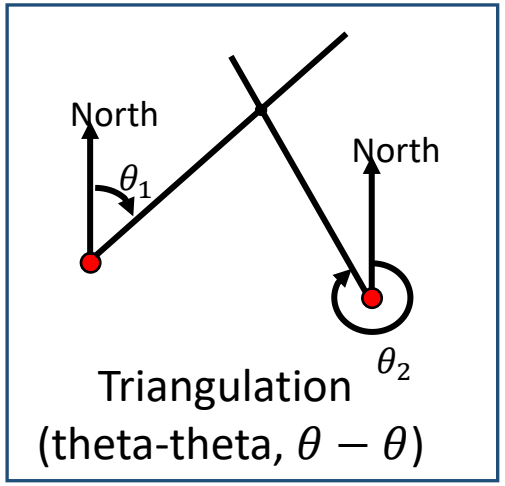
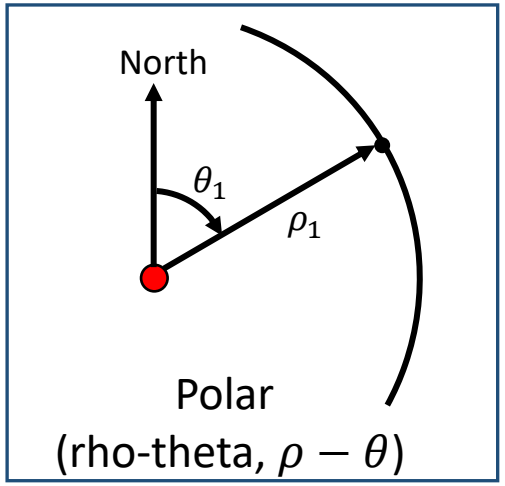
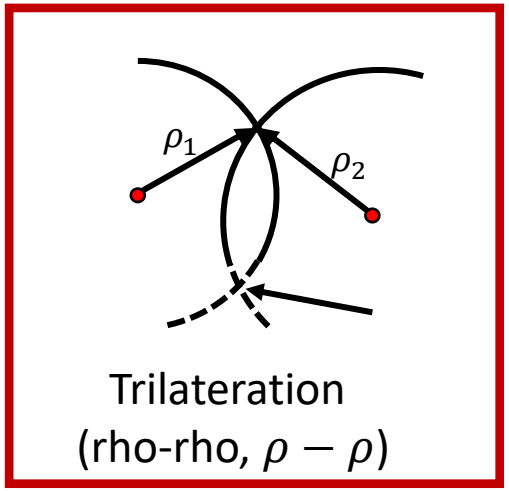
Position Determination



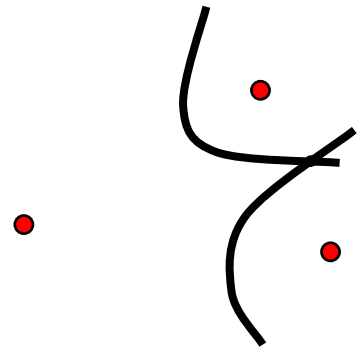
Triangulation
(theta-theta, $\theta - \theta$)



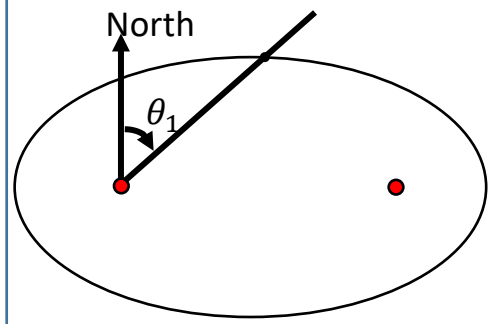
Polar
(rho-theta, $\rho - \theta$)



Trilateration
(rho-rho, $\rho - \rho$)



Hyperbolic/elliptical



Combination

So, how about GNSS?

*Position Determination is also known as position fixing

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Two-Dimensional $\rho - \rho$ Positioning

Foghorn Example

Source: wikipedia

A foghorn or fog signal is a device that uses sound to warn vehicles of navigational hazards such as rocky coastlines, or boats of the presence of other vessels, in foggy conditions

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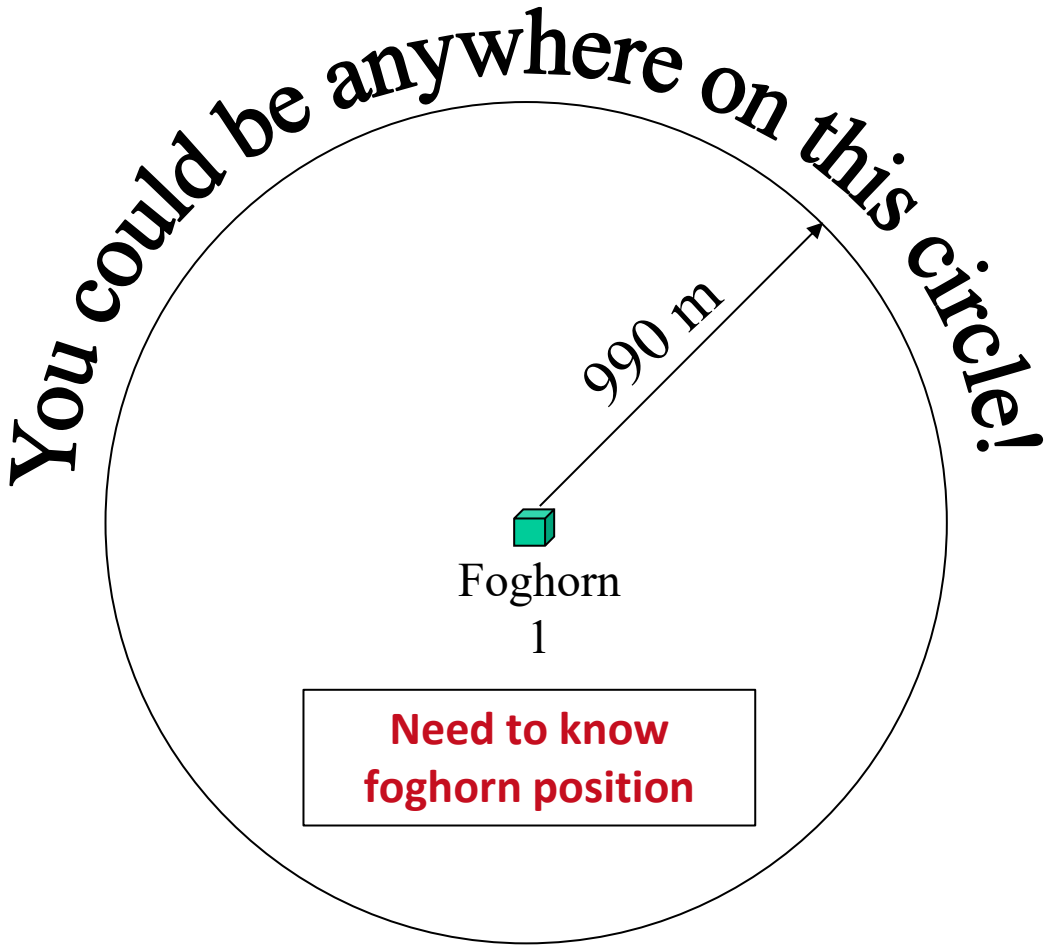
Two-Dimensional $\rho - \rho$ Positioning

- Foghorn sends signal at **12:00:00**
  - **Time-of-Transmission** (**TOT**)
- Receiver receives message at **12:00:03**
  - **Time-of-Arrival** (**TOA**) or **Time-of-Reception** (**TOR**)
- Speed of sound:
  - v_{sound} = 330m/s
- Range between you and the foghorn (#1) is:

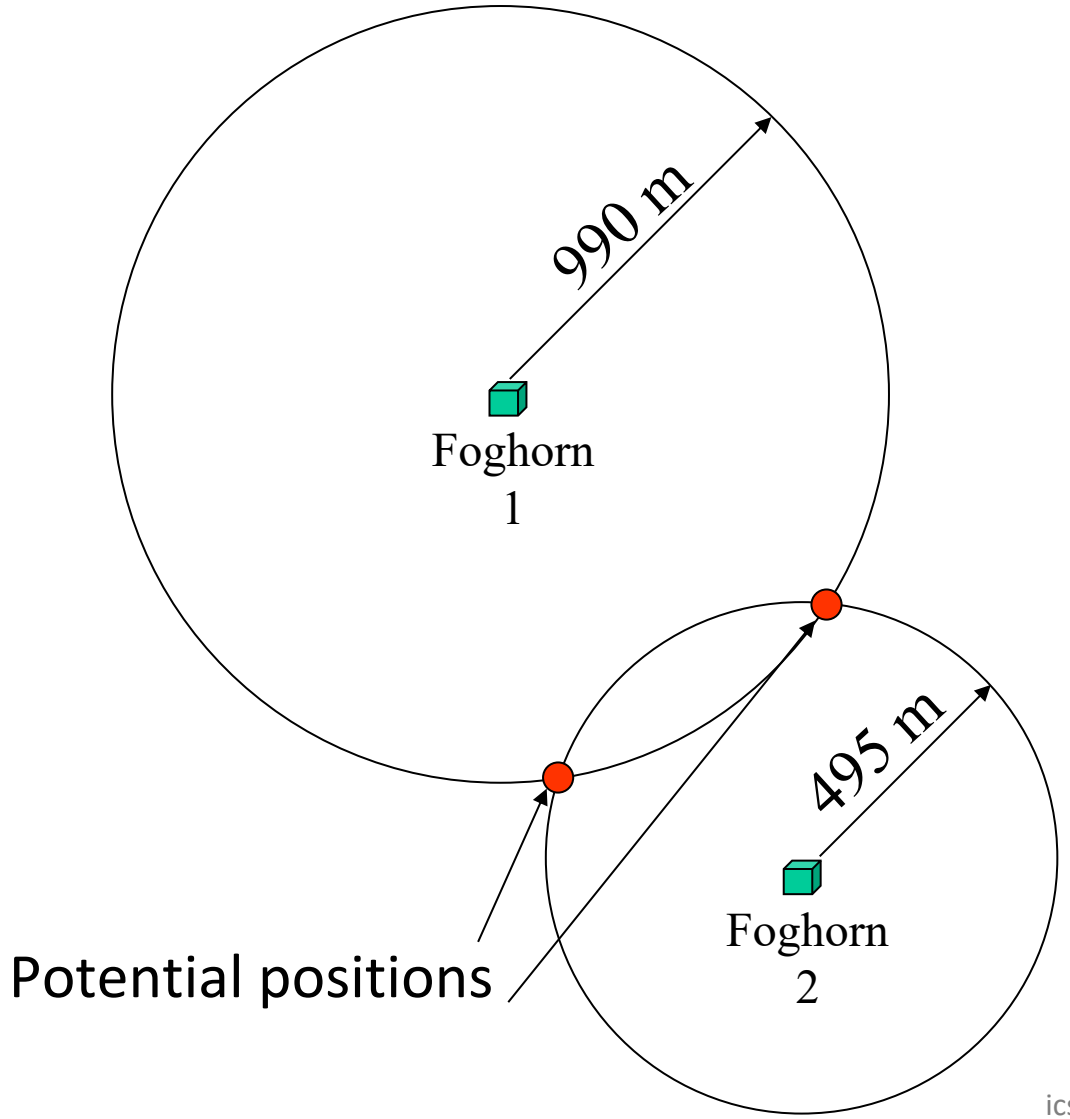
$$R = v_{sound}(\mathrm{TOR} - \mathrm{TOT}) = v_{sound}\Delta t = 990m$$

- Unable to determine exact position in this case



Two-Dimensional $\rho - \rho$ Positioning

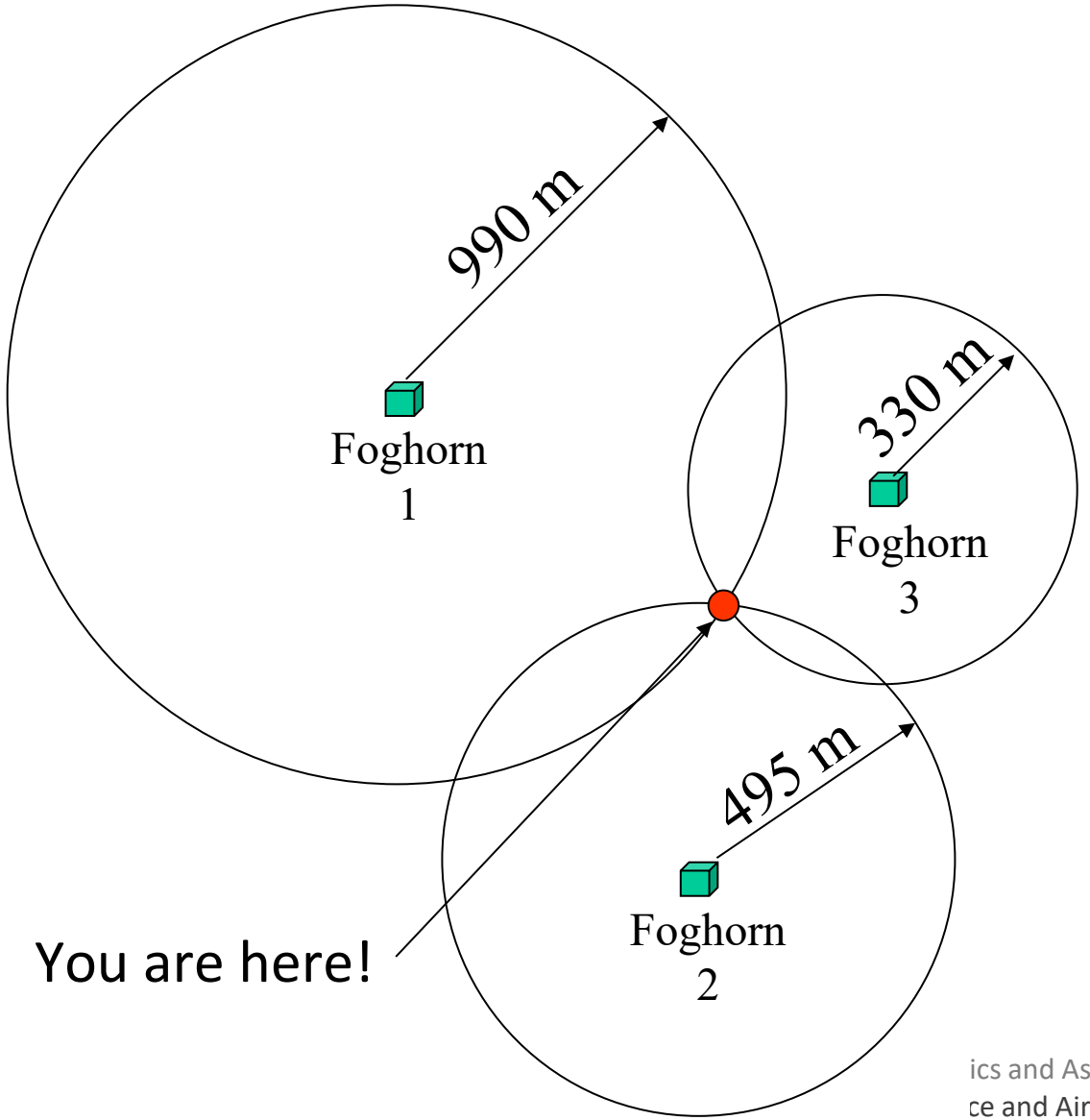
- Now, you take a measurement from foghorn #2 at TOA = **12:00:01.5** (for a range of **495 m**)
- Yields two potential solutions
  - How would you determine the correct solution?



Two-Dimensional $\rho - \rho$ Positioning

- You get a third measurement from foghorn #3 at TOA = **12:00:01**
- Range = **330 m**
  - Now there's a unique solution

Important:
Clocks of all Foghorns must be synchronized and their locations known



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Receiver Clock Errors

- The foghorn example assumed that the foghorn "**receiver**" had a perfectly synchronized clock, so the measurements were perfect
- What happens if there is an **unknown receiver clock error** between the receiver clock and the Foghorn clocks?
- Effect on range measurement
  - Without clock error

$$R = v_{sound}(TOA - TOT) = v_{sound}\Delta t$$

  - With clock error δt

$$PR = v_{sound}(TOA + \delta t\ - TOT) = v_{sound}(\Delta t + \delta t)$$

Range with error (referred to as: **pseudo-range**)

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Receiver Clock Errors – 1D Example

- Now, we will look at the foghorn example, except in only one dimension
  - The foghorn(s) and receiver are constrained to be along a line
  - We want to determine the position of the receiver on that line

Foghorn 1

- If the receiver measured a signal at **12:00:10**, where is it on the line?
- Now, assume an unknown clock bias δt in the clock used by the foghorn receiver
- Your foghorn receiver measures a foghorn blast at **12:00:10**
- What can you say about where you are?

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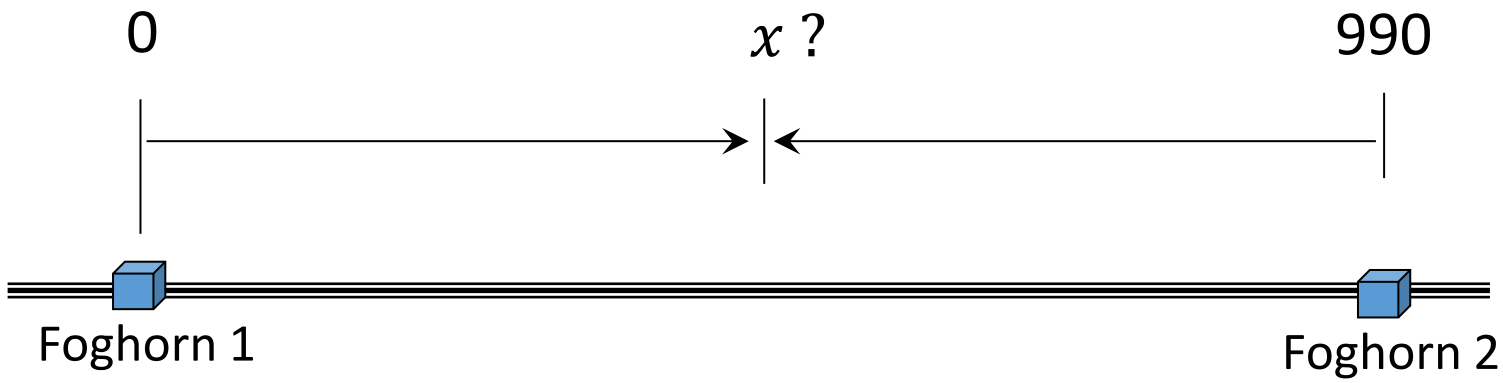
Receiver Clock Errors – 1D Example

- Clearly, more information is needed
- Assume that there is a second foghorn located 990 m away from the first

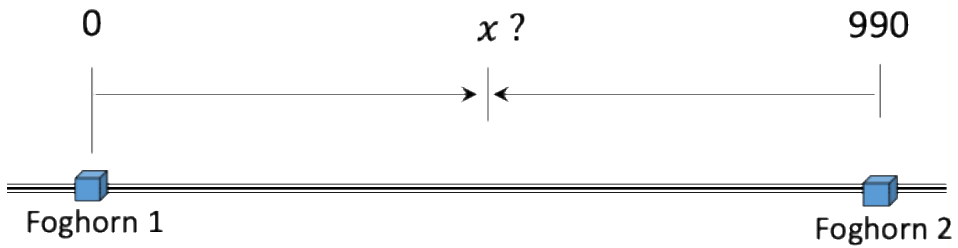


- You receive a signal from the second foghorn at **12:00:09**
- What can you tell about where you are at this point?

Receiver Clock Errors – 1D Example

- Here are the **pseudorange measurements** we have:

$$PR_1 = v_{sound} \times 10 = 330 \times 10 = 3300$$
$$PR_2 = v_{sound} \times 9 = 330 \times 9 = 2970$$



- From the **pseudorange** equation:

$$PR_1 = v_{sound}(\Delta t_1 + \delta t) = R_1 + v_{sound}\delta t = x + v_{sound}\delta t = 3300$$
$$PR_2 = v_{sound}(\Delta t_2 + \delta t) = R_2 + v_{sound}\delta t = 990 - x + v_{sound}\delta t = 2970$$

- Rearranging terms we get

$$x + v_{sound}\delta t = 3300$$
$$x - v_{sound}\delta t = -1980$$

- We can then solve for the two unknowns

$$\delta t = 8\ seconds$$
$$x = 660m$$

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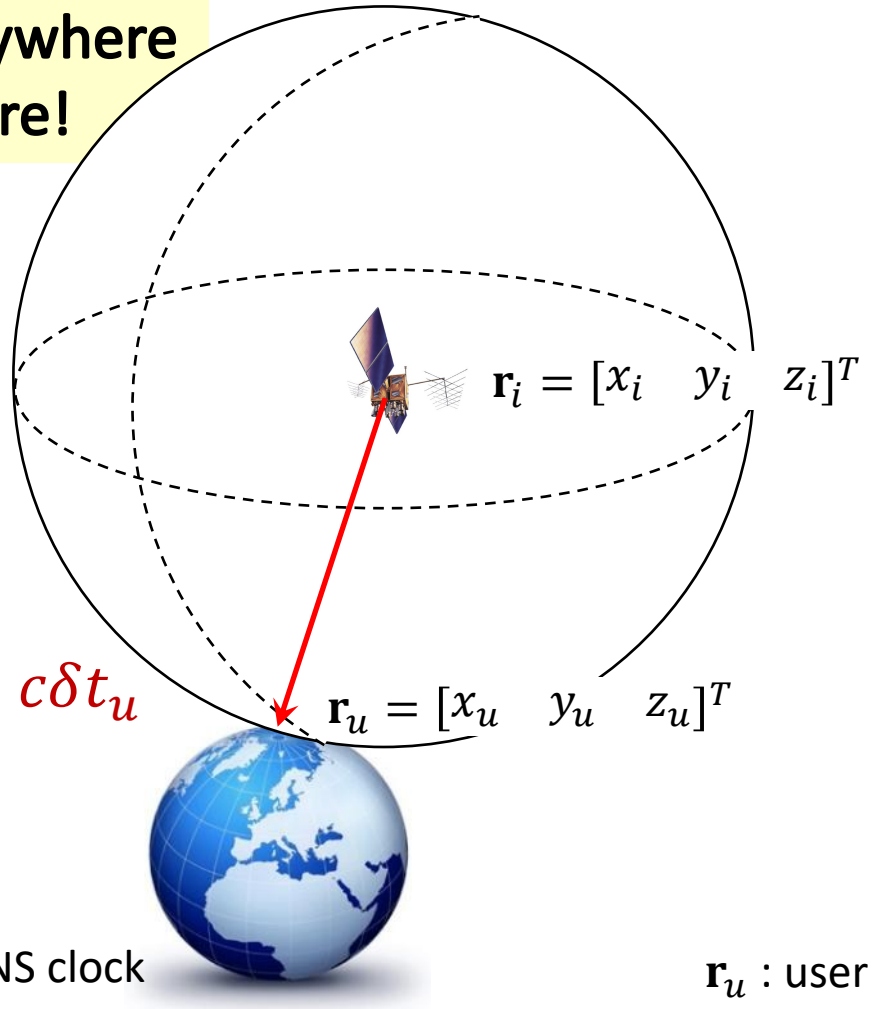
GNSS – from 2D to 3D

You could be anywhere on this sphere!



Foghorn → GNSS Satellite

Circles → Spheres

Speed of sound → Speed of light

Pseudorange:

$$PR_i = \sqrt{(x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2} + c\delta t_u$$

$$= \|\mathbf{r}_u - \mathbf{r}_i\| + c\delta t_u$$

Euclidean distance between user and satellite position

Difference between the user clock and the GNNS clock

$\mathbf{r}_u$: user position

$\mathbf{r}_i$: satellite 'i' position

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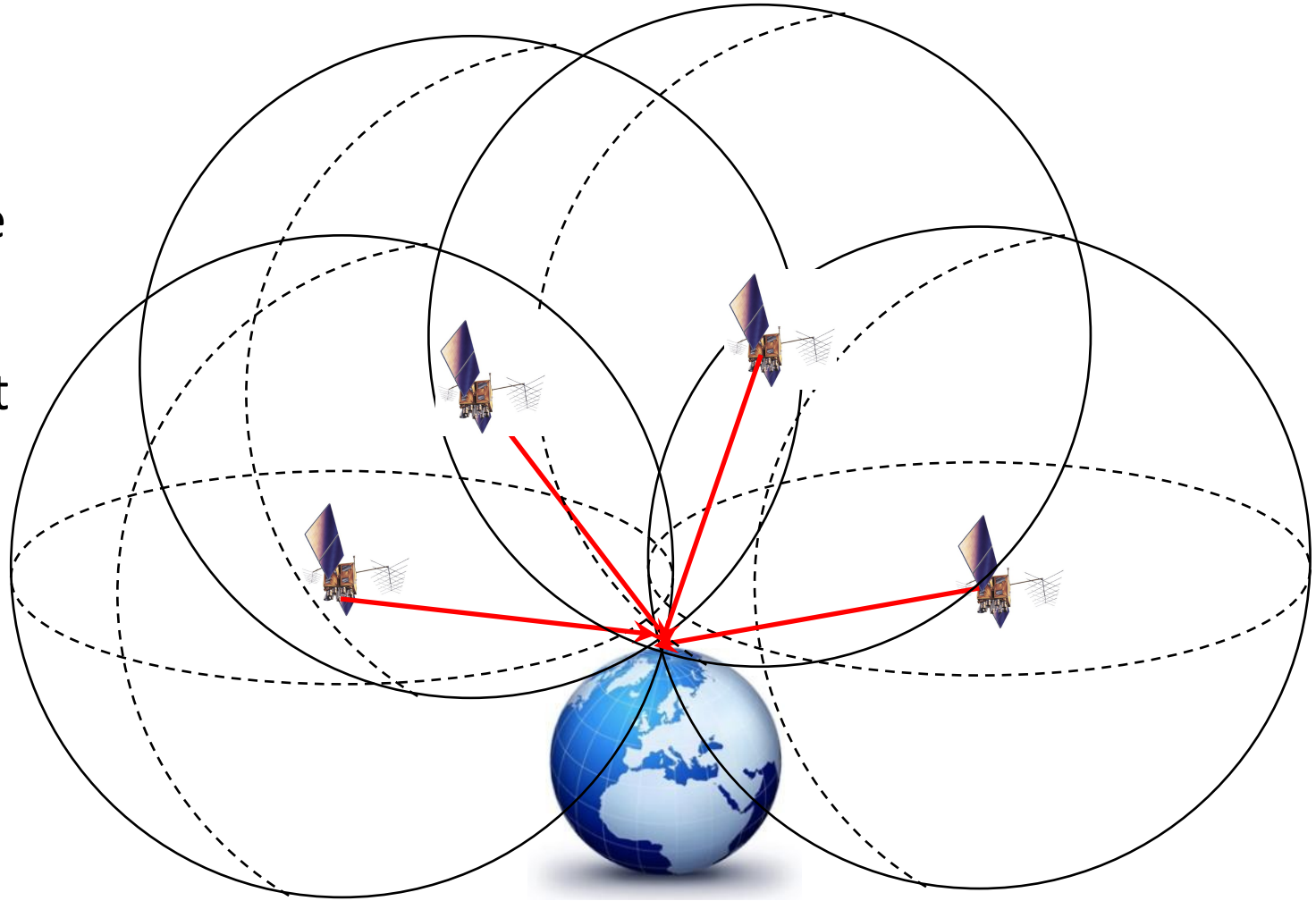
GNSS – from 2D to 3D

Foghorn → GNSS Satellite

Circles → Spheres

Speed of sound → Speed of light

User is on the intersection of at least 4 spheres



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Receiver Clock Errors – Extend to 3D

- In the **single-dimensional** case
  - We needed two measurements to solve for the two unknowns, x and δt.
  - The quantities x and $(990 - x)$ were the "distances" between the position of the receiver and the two foghorns.
- In **three-dimensional** case
  - We need four measurements to solve for the four unknowns, x, y, z and δt.
  - The distances between receiver and satellite are not linear equations (as was case in single-dimensional case).
  - The >four equations to be solved simultaneously, for **pseudorange** measurements $PR_1 \ldots PR_N$ and **transmitter positions** $(x_1, y_1, z_1) \ldots (x_N, y_N, z_N)$

$$PR_j = \sqrt{(x_u - x_j)^2 + (y_u - y_j)^2 + (z_u - z_j)^2} + c\delta t_u$$

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Approaches to solve these equations

- Closed form solutions
  - Complicated
  - May not give as much insight
- Iterative techniques based on linearization
  - Often using least-squares estimation
  - Arguably the simplest approach
- Kalman filtering
  - Similar to least-squares approach, except with additional ability to handle measurements over a period of time

Ordinary Least Squares Solution

$$PR_j = \sqrt{(x_u - x_j)^2 + (y_u - y_j)^2 + (z_u - z_j)^2} + c\delta t_u$$

$$\Delta PR_j = \frac{\partial PR_j}{\partial x_u}\Delta x_u + \frac{\partial PR_j}{\partial y_u}\Delta y_u + \frac{\partial PR_j}{\partial z_u}\Delta z_u + c\delta t_u$$

$$\Delta PR_j = \frac{x_u - x_j}{R_j}\Delta x_u + \frac{y_u - y_j}{R_j}\Delta y_u + \frac{z_u - z_j}{R_j}\Delta z_u + c\delta t_u$$

$$\Delta\mathbf{PR} = \begin{bmatrix} \frac{x_u - x_1}{R_1} & \frac{y_u - y_1}{R_1} & \frac{z_u - z_1}{R_1} & 1 \\ \frac{x_u - x_2}{R_2} & \frac{y_u - y_2}{R_2} & \frac{z_u - z_2}{R_2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_u - x_N}{R_N} & \frac{y_u - y_N}{R_N} & \frac{z_u - z_N}{R_N} & 1 \end{bmatrix} \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \\ c\delta t_u \end{bmatrix} = \mathbf{H}\Delta\mathbf{x}$$

$$\Delta\hat{\mathbf{x}} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\Delta\mathbf{PR}$$

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_k + \Delta\hat{\mathbf{x}}$$

Various Systems

System	Source	Coding	Frequencies	Status (1 February 2026)
Global Positioning System (GPS)	United States	CDMA	L1 (1575.42MHz) L2 (1227.60MHz) L5 (1176.45MHz)	Operational (28)
GLONASS	Russia	FDMA/CDMA	FDMA (1602 MHz) FDMA (1246 MHz) CDMA (1600.995MHz) CDMA (1248.06MHz) CDMA (1202.25MHz)	Operational (24 - nominal)
Galileo	European Union	CDMA	E1 (1575.420MHz) E6 (1278.750MHz) E5 (1191.795MHz) E5a (1176.450MHz) E5b (1207.140MHz)	Operational (26)
BeiDou	China	CDMA	B1 (1561.098MHz) B1-2 (1589.742MHz) B2 (1207.140MHz) B3 (1268.52MHz)	Operational (30~35)

Status info:
GPS: https://www.navcen.uscg.gov/gps-constellation
Galileo: https://www.gsc-europa.eu/system-service-status/constellation-information
GLONASS: https://glonass-iac.ru/en/GLONASS/

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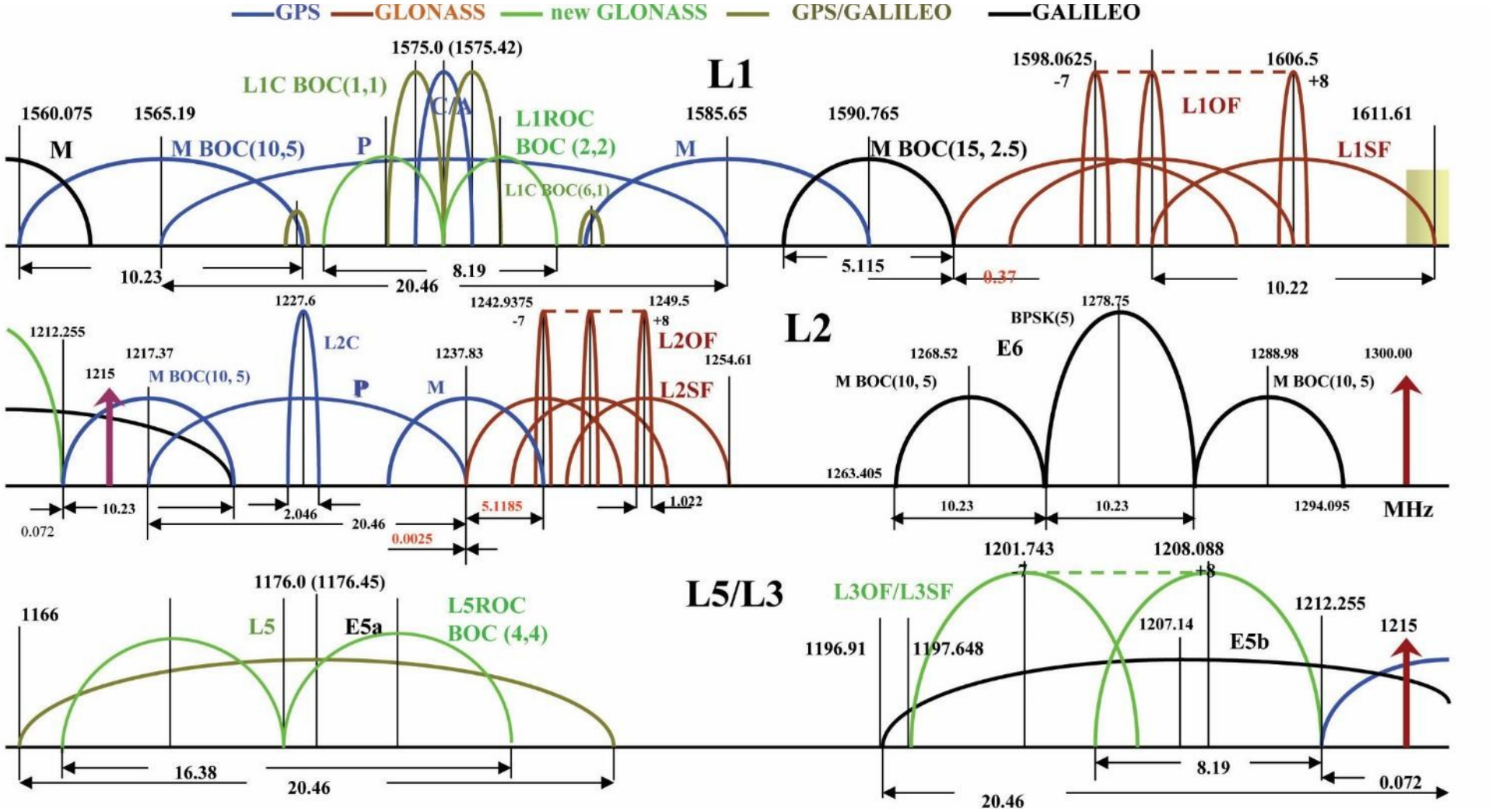
Various Systems



From: http://www.insidegnss.com/node/648

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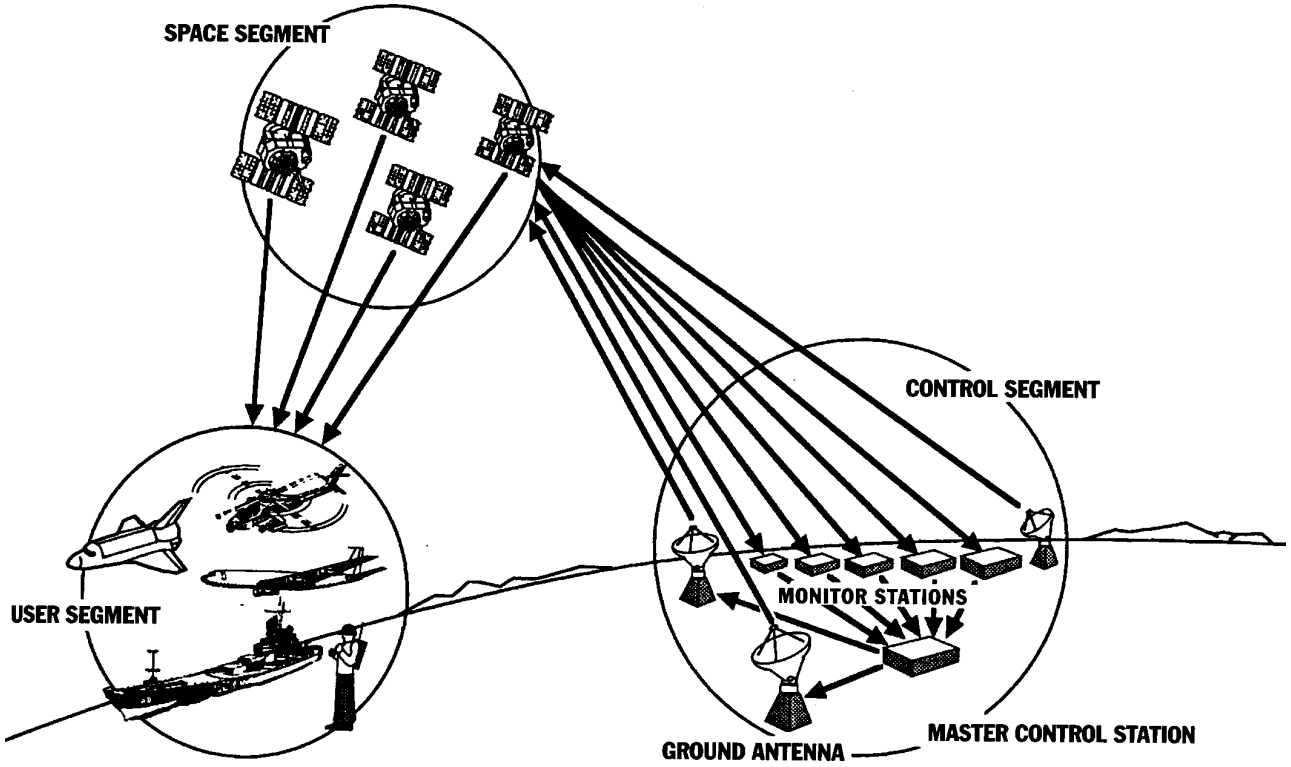
Three Segments of a GNSS System



- **Space**: the satellites themselves
- **Control**: monitors and controls satellites, generates ephemeris (and other) data
- **User**: anything that receives the GPS signal

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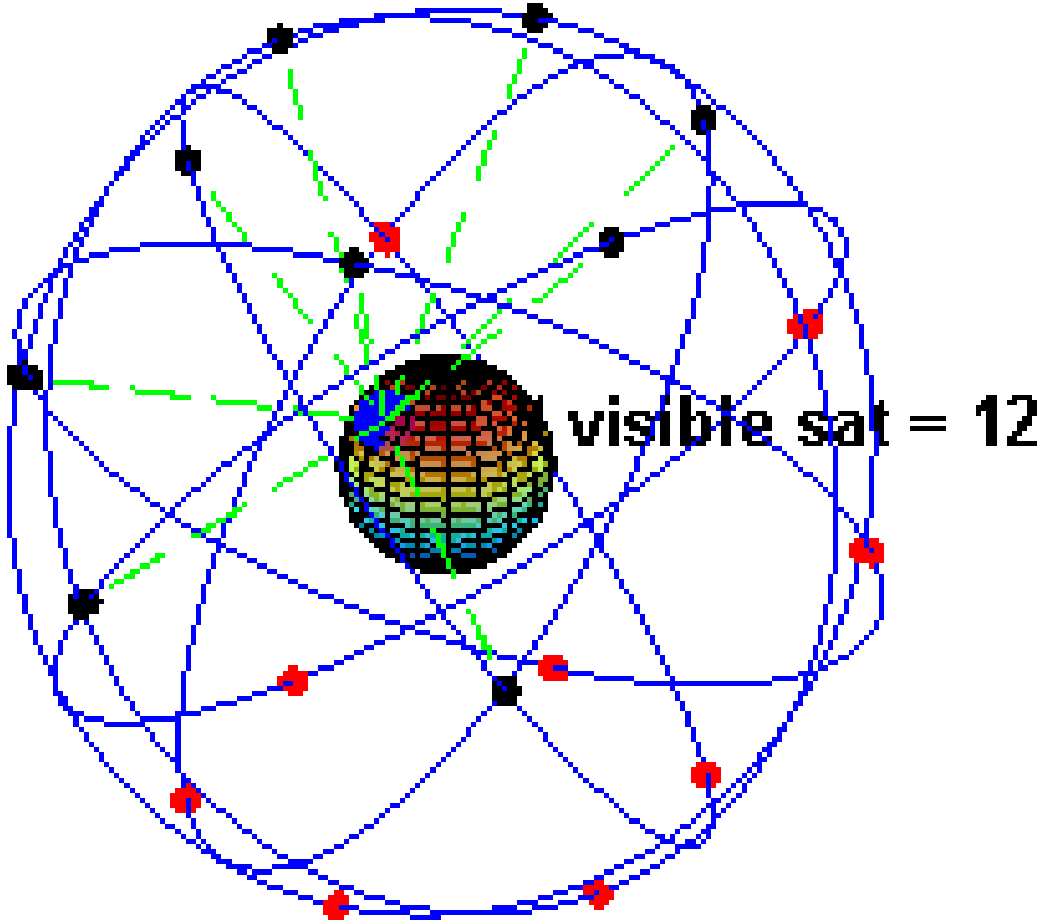
Global Positioning System (GPS) Example



http://en.wikipedia.org/wiki/File:ConstellationGPS.gif

- 24 Satellites in 6 orbital planes inclined at 55 degrees with respect to the equatorial plane;
- Currently 26 satellites
- Repeating ground tracks - 23 hr 56 min;
- At least 4 satellites in view to a user;
- Anywhere on the Earth (7.5 on the average);
- Continuous, all-weather coverage;
- Orbit altitude is ~20,200 km (10,900 nmi) relative to the earth surface.

http://www.navcen.uscg.gov/
http://gps.gov

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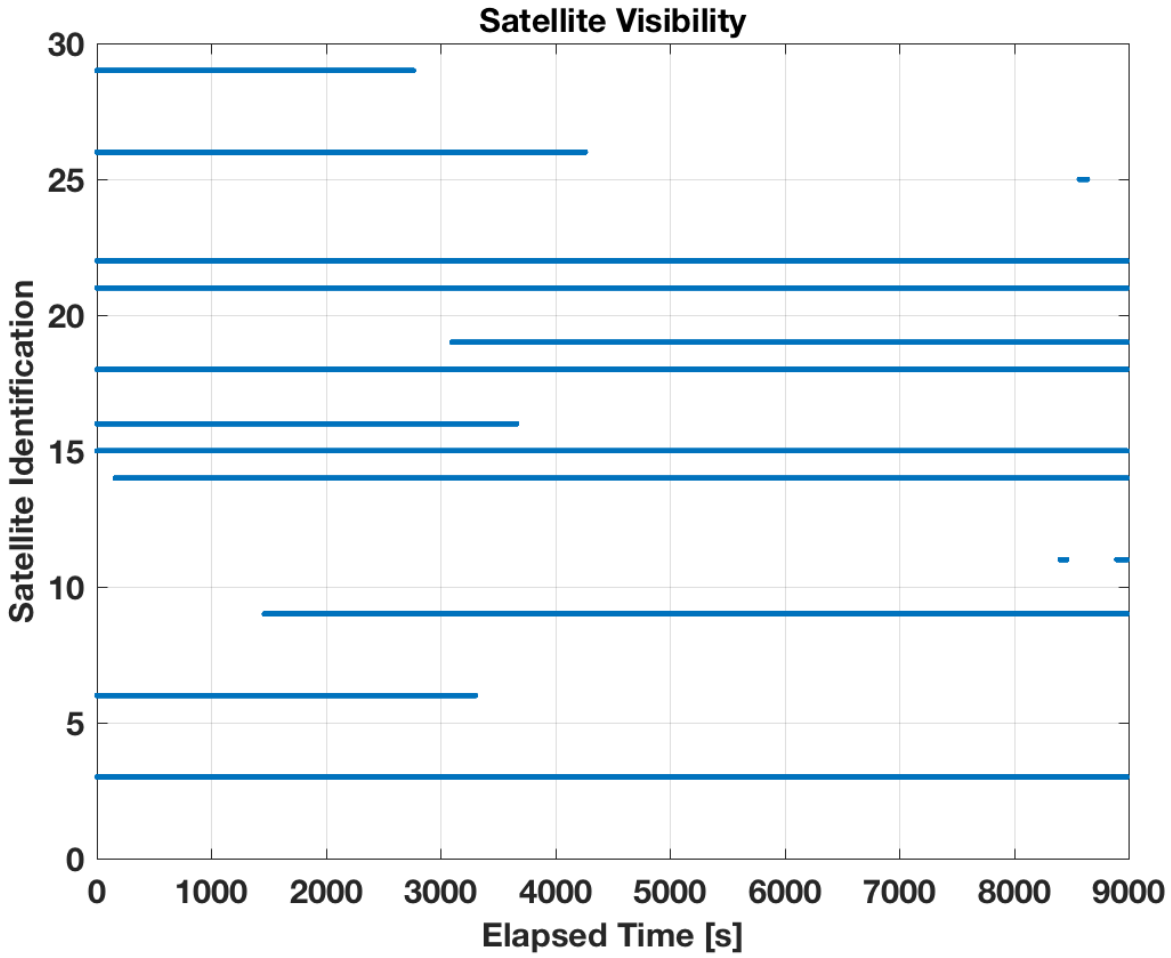
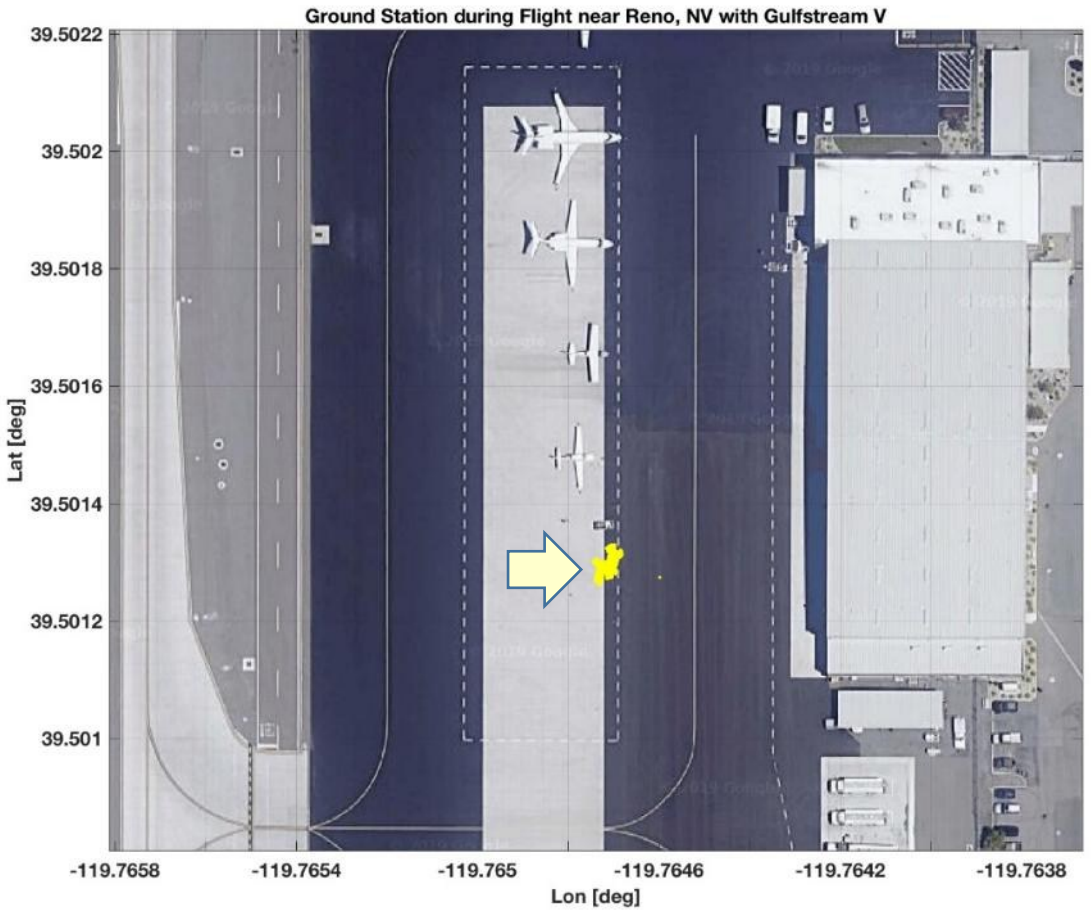
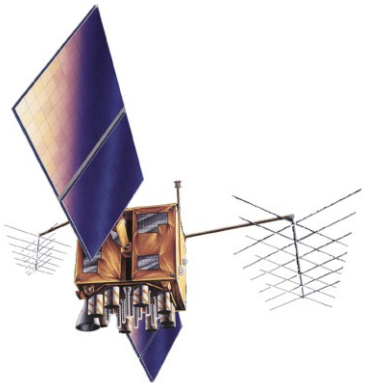
Satellite Visibility





Real stationary data collected during a flight test with a Gulfstream V at Reno, NV

GNSS Signal Structure – General Expression



$$s_{tx,j}(t) = A_{tx,j} G_j(t) D_j(t) \sin(2\pi f_{tx} t + \alpha_j)$$

Carrier-frequency

Transmitted signal

Transmitted signal amplitude

represents the transmitted spreading code, used to synchronize the received and locally generated signal (in receiver)

Navigation message data

Carrier signal

SV Carrier-Phase Offset

GNSS Control Segment

$$PR_j = \sqrt{(x_u - x_j)^2 + (y_u - y_j)^2 + (z_u - z_j)^2} + \delta t_u$$

- Manages constellation (flies the satellites)
- Monitors GNSS system performance
- Calculates data sent over the **navigation message**
- **Navigation message contains parameters (trajectory model) so you can determine where the satellite is/was**

$$s_{tx,j}(t) = A_{tx,j} G_j(t) D_j(t) \sin(2\pi f_{tx} t + \alpha_j)$$

- Ground Control station(s) communicates with satellite using, for example, S-band data link (intermittent)
- Other name for satellites: **Space Vehicles** (SV)

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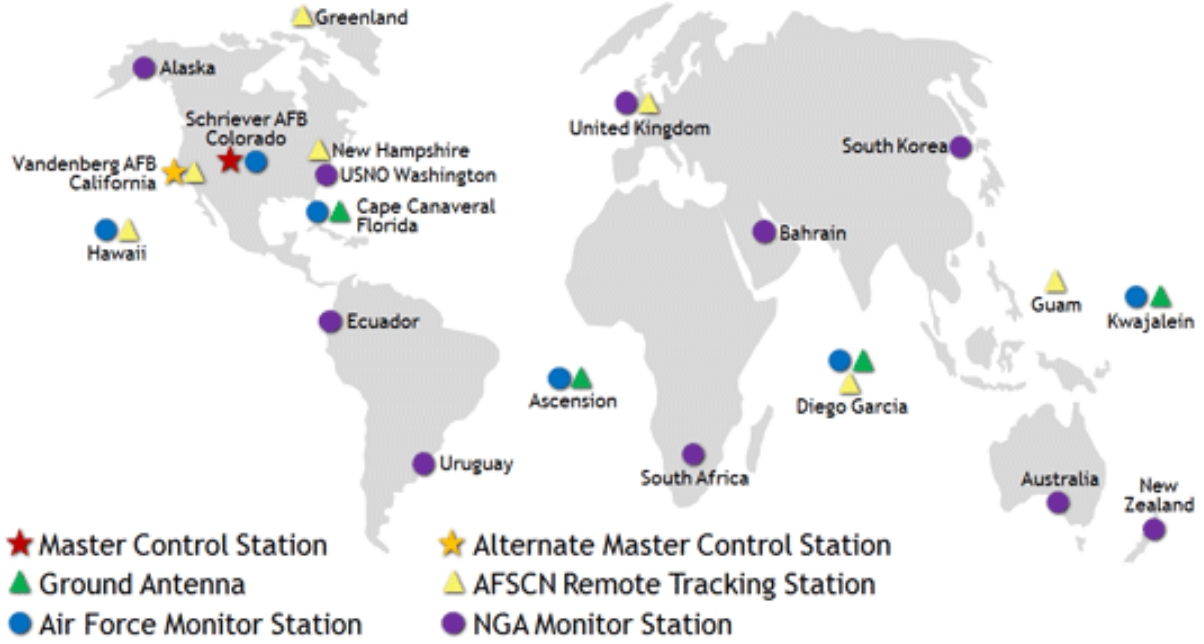
GPS Control Segment Example



http://www.gps.gov

https://gssc.esa.int/navipedia/index.php?title=File:Galileo_Global_Ground_Segment.jpg

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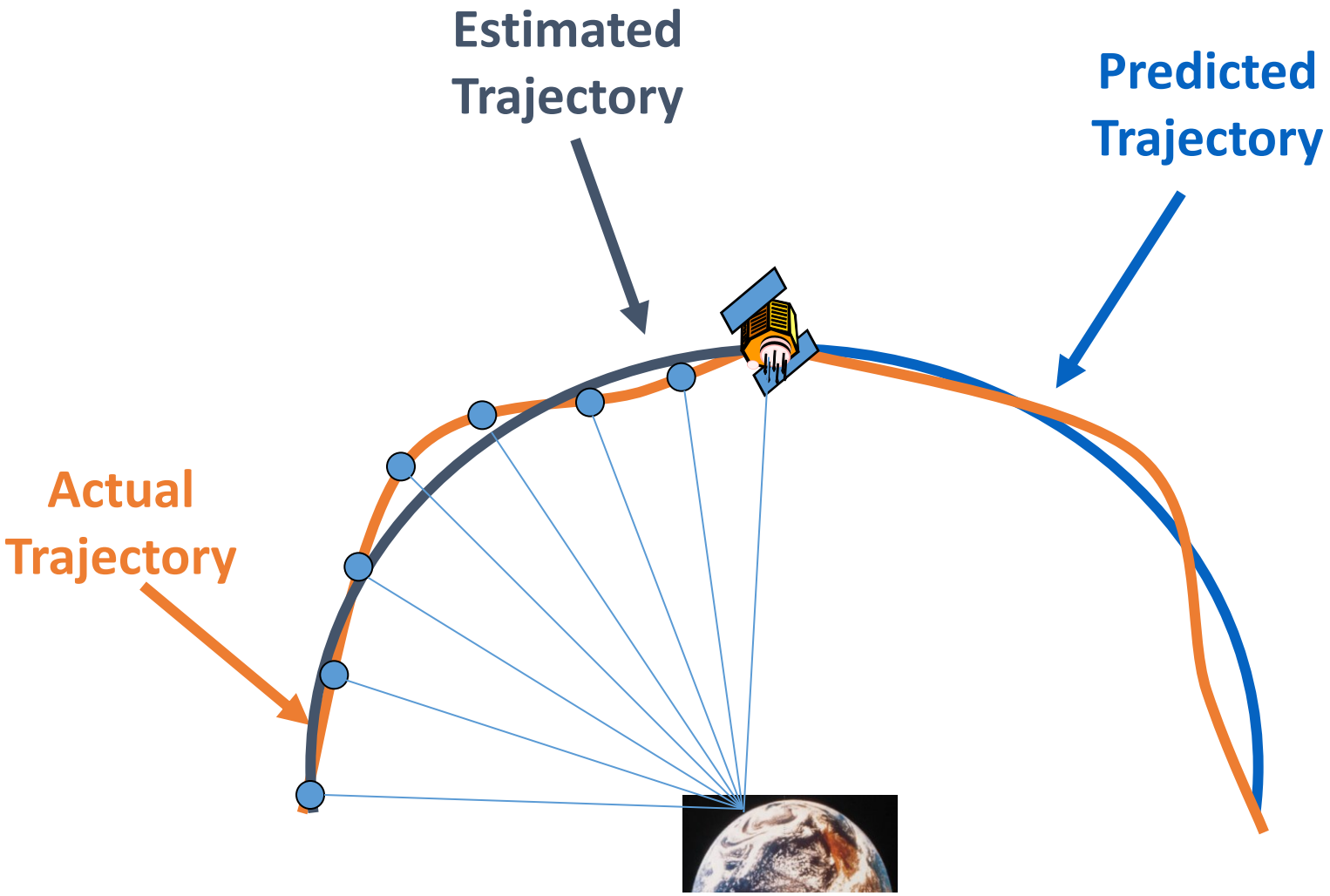
Satellite Trajectory Estimation and Prediction



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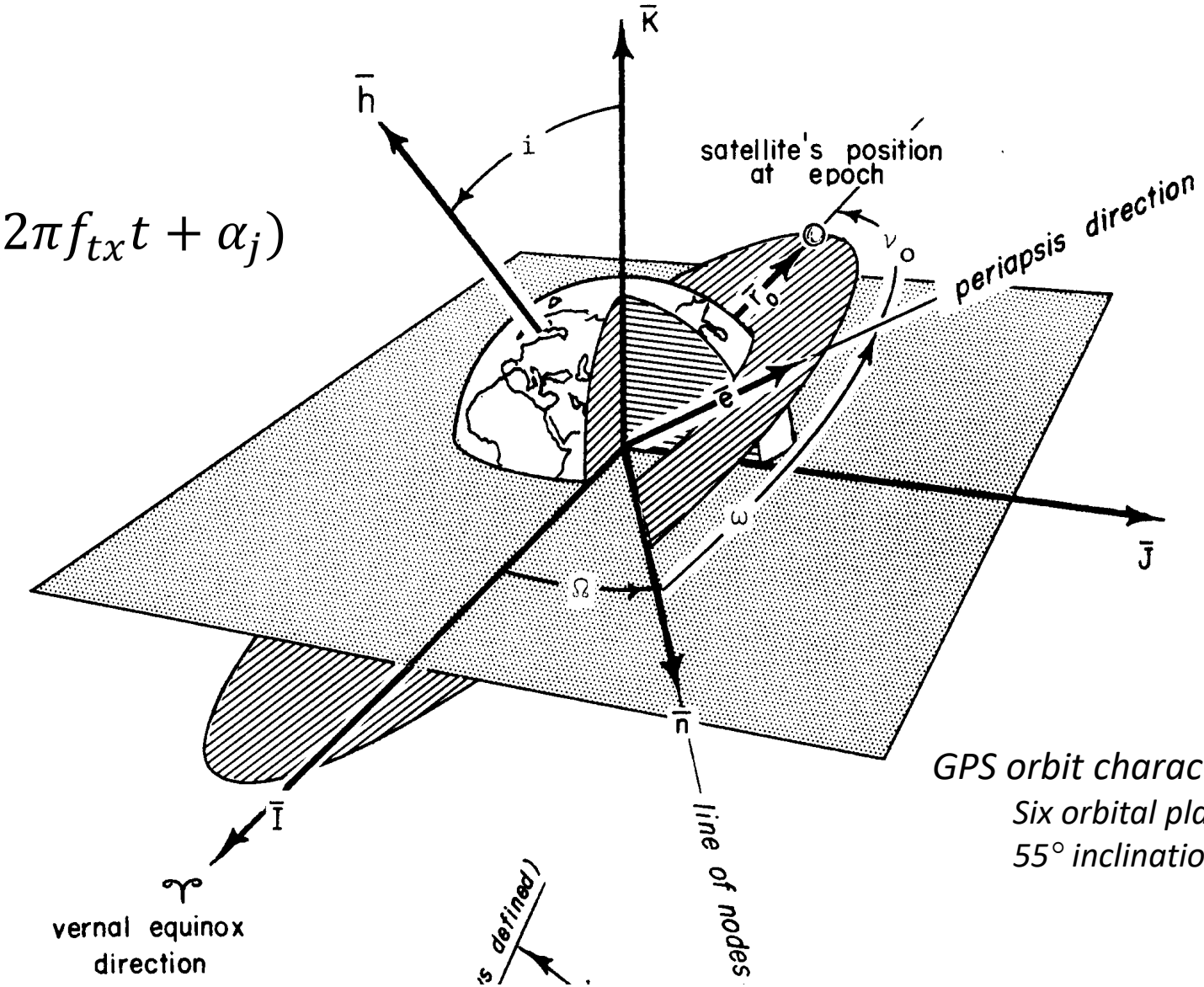
GNSS Ephemeris Data – Parameters

$$s_{tx,j}(t) = A_{tx,j} G_j(t) D_j(t) \sin(2\pi f_{tx} t + \alpha_j)$$



GPS orbit characteristics:
Six orbital planes
55° inclination angle

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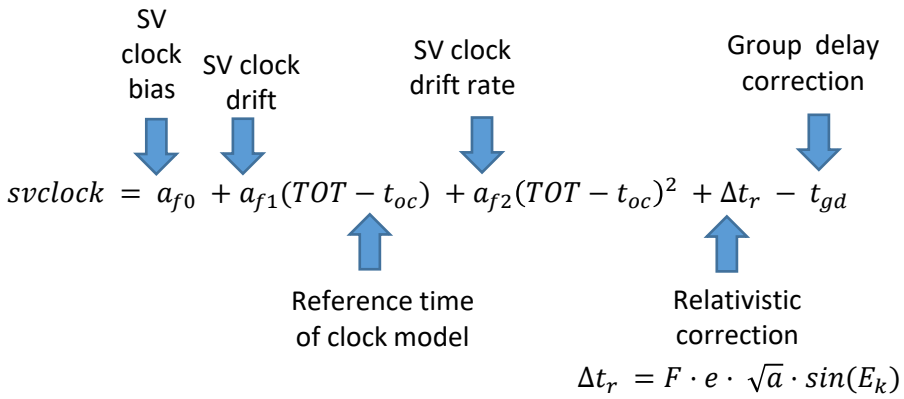
GNSS Ephemeris Data – Clock Model

- Even though the GNSS satellites are clock synchronized, each satellite's clock deviates just a little bit from GNSS time.
- This difference must be corrected; therefore, the ephemerides also include clock model parameters:
  - A clock bias term
  - A clock drift and drift rate term
  - A relativity correction (since the in-orbit clock is accelerating)
  - A group delay due to delays in the instrumentation



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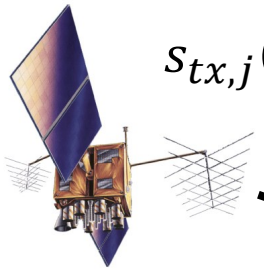
GNSS Received Signal Power (Link Budget)

$$s_{tx,j}(t) = A_{tx,j} G_j(t) D_j(t) \sin(2\pi f_{tx} t)$$



Transmit
For example, GPS L1:
478.63 W = 26.8 dBW

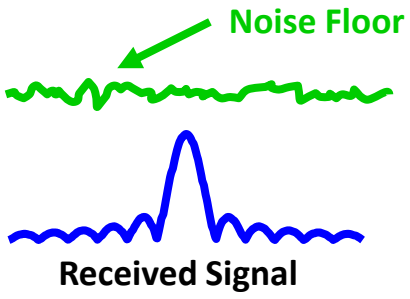
line-of-sight

Free Space Loss: -182.4 dB

Atmospheric Loss: -4.4 dB

Total loss: -186.8 dB (= $10^{-18.68}$)

Thermal Noise
(2 MHz Bandwidth):
10^{-14} W = -140 dBW



Received Signal:
10^{-16} W = -160 dBW

GNSS Receiver

- Power in dBW = $10\log_{10}$ (Power in W)
- Power in dBm = $10\log_{10}$ (Power in mW)
- 1 mW = 0.001 W = 0 dBm = -30 dBW

Institute of Aeronautics and Astronautics
Chair of Flight Guidance and Air Transport

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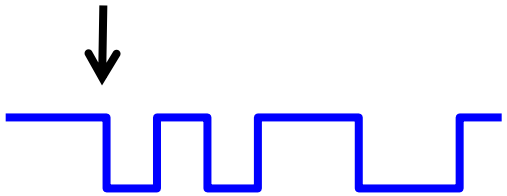
Timing in Signal

Transmitter side:

Time-of-Transmission (TOT) is encoded on the signal (signal contains a copy of the time-of-transmission in navigation data)

Receiver (user) side:

Time-of-Transmission (TOT)



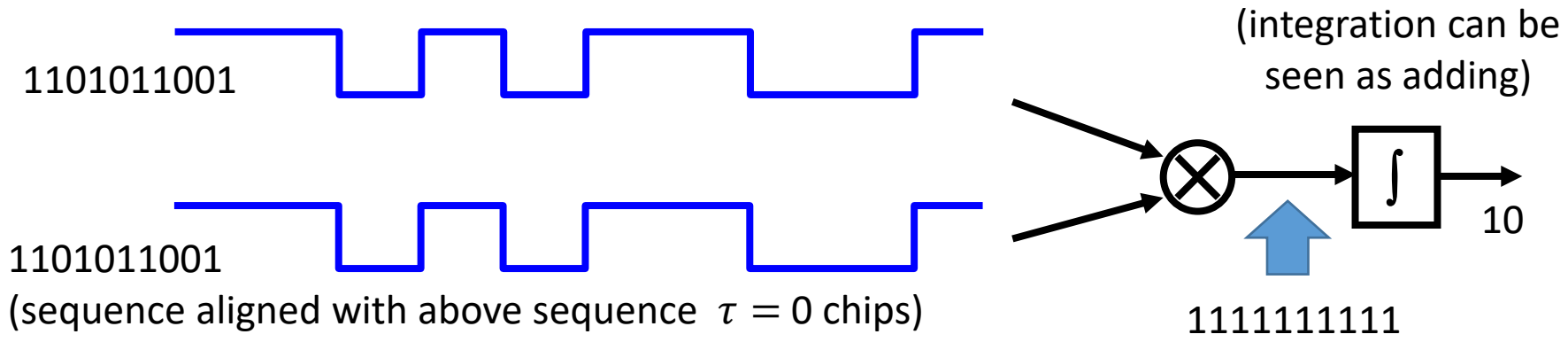
Estimated Range = (TOR – TOT)*(speed of the signal)

Time-of-Reception (TOR)

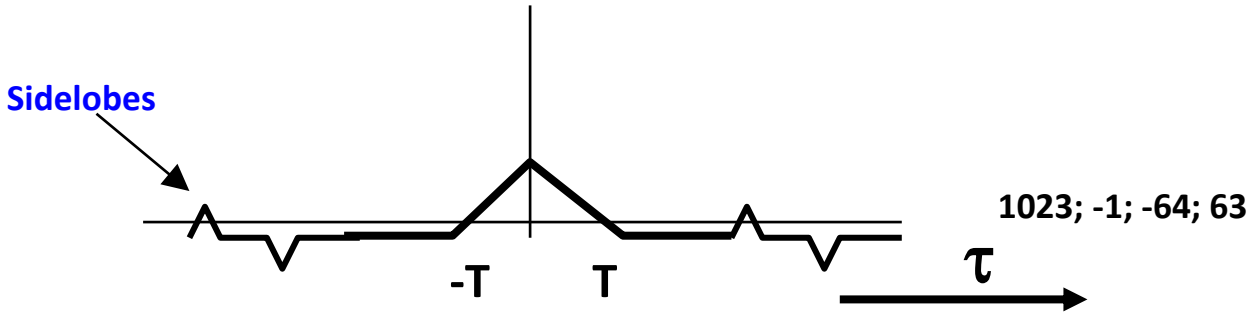
To find the TOT accurately, you need to know where the TOT is located by synchronizing the received signal with a local copy

Synchronization using Correlators

- Example:



- For GPS PRN code (for various alignments, τ)



- Correlation is maximum when the two PRNs are aligned

GPS Cross-correlation Animation

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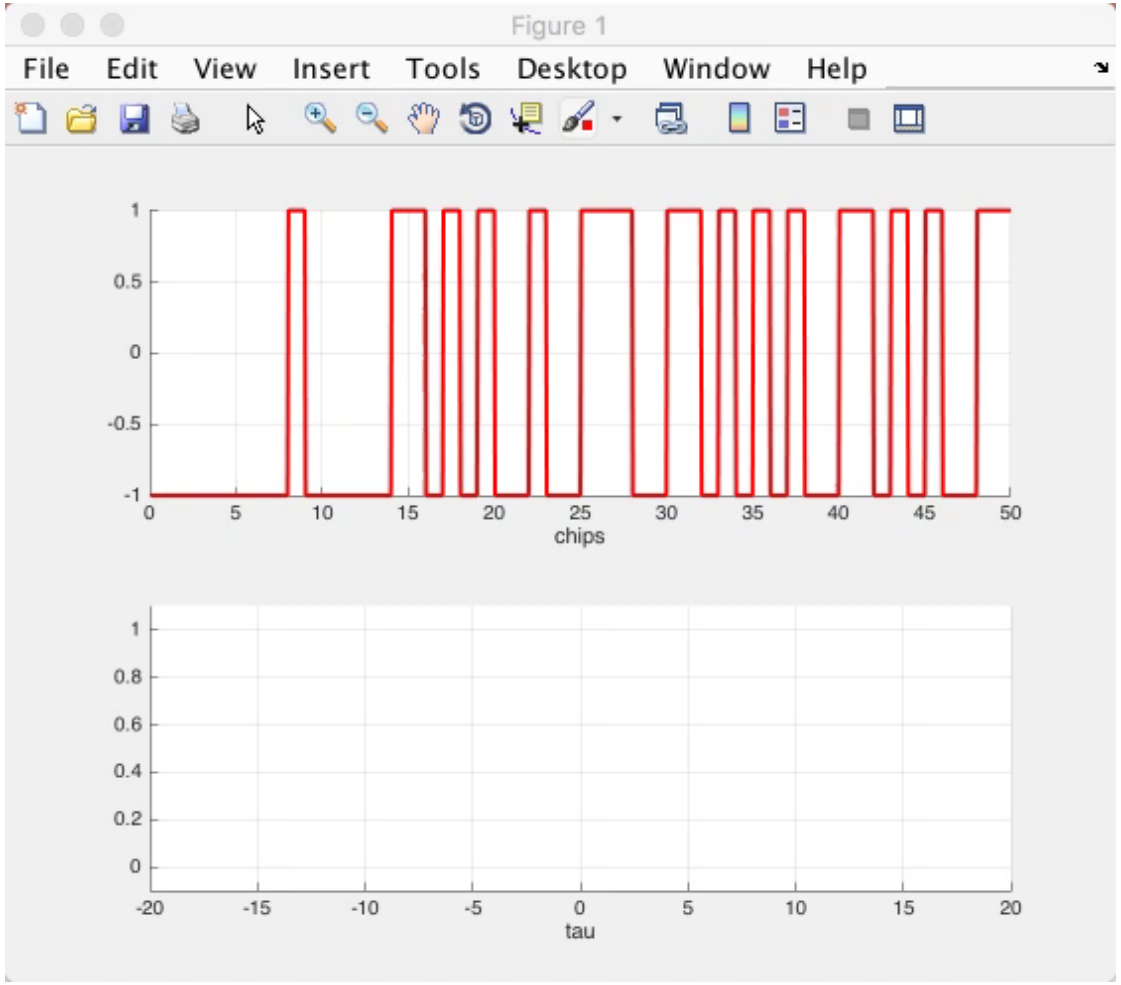

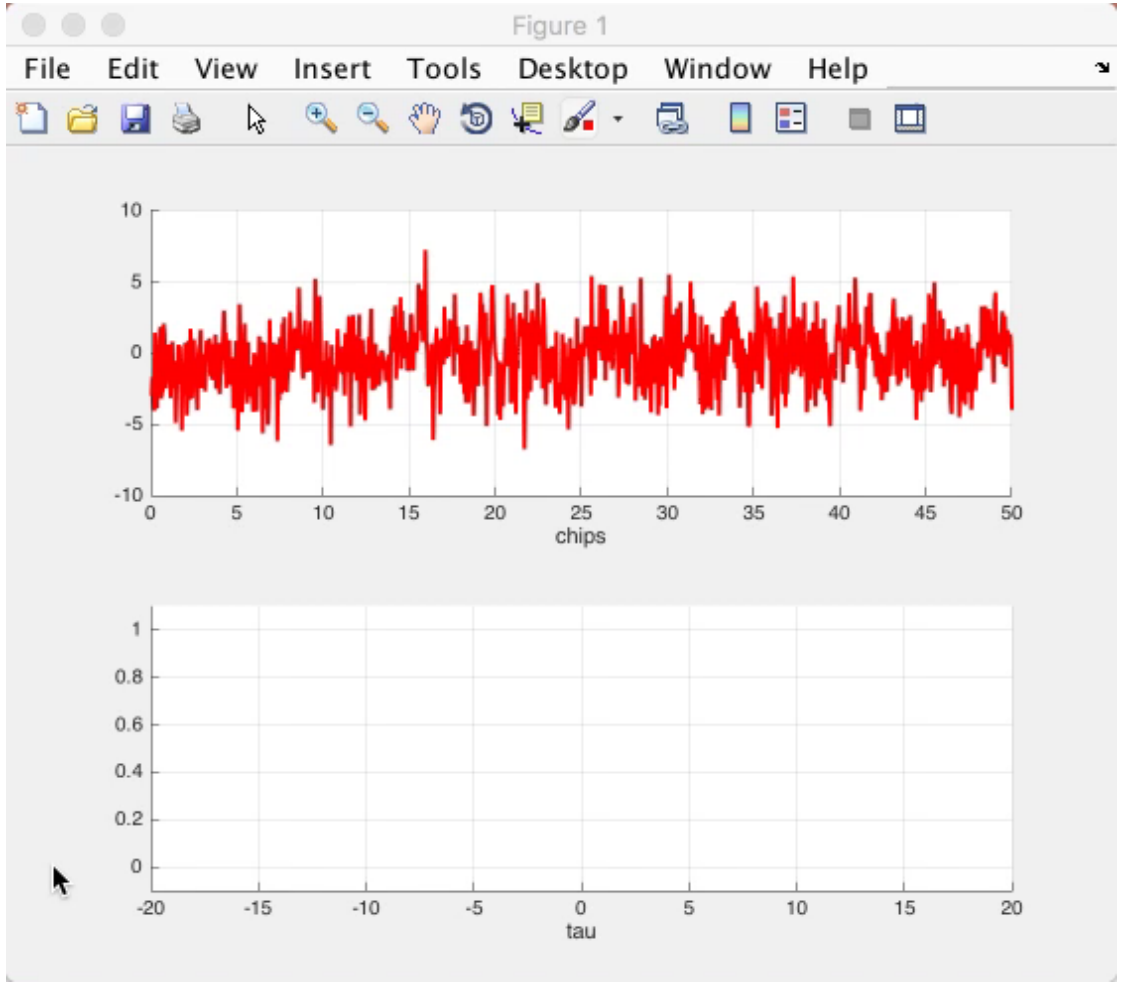

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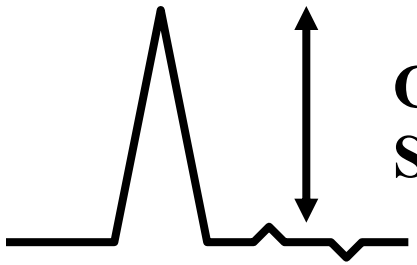
GNSS Cross-correlation

- Cross-correlation is the key to GNSS operation:
  - The received signal is below the noise floor.
  - The correlator allows the receiver the align a locally-generated code with the incoming code.
  - Integration over all chips increases the received signal power.
    The noise; however, is reduced.



GPS example: Correlation
Sidelobes are at least 22dB down

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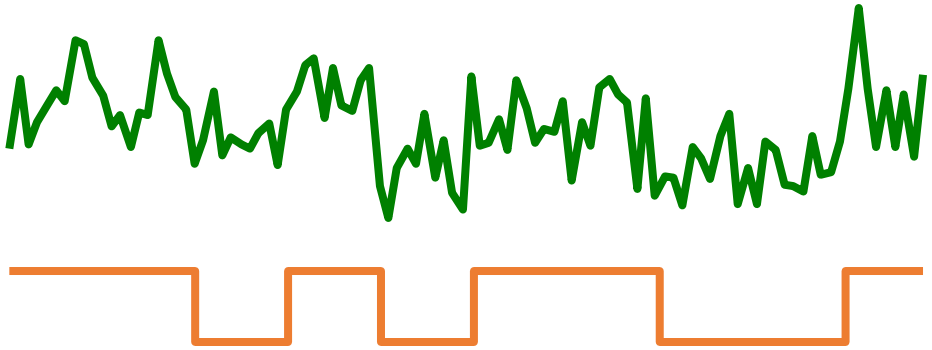
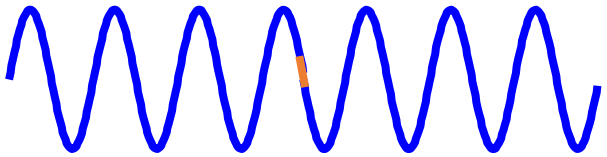
“Raw” Measurements

- Code-phase measurement or pseudorange (e.g., C/A)



- Carrier-phase measurement or integrated Doppler frequency shift (also referred to as accumulated Doppler shift)



	Pseudorange	Carrier-Phase
Type of measurement	Range (absolute)	Range (ambiguous)
Measurement precision	dm-level	mm-level
Robustness	More robust	Less robust (cycle slips possible)

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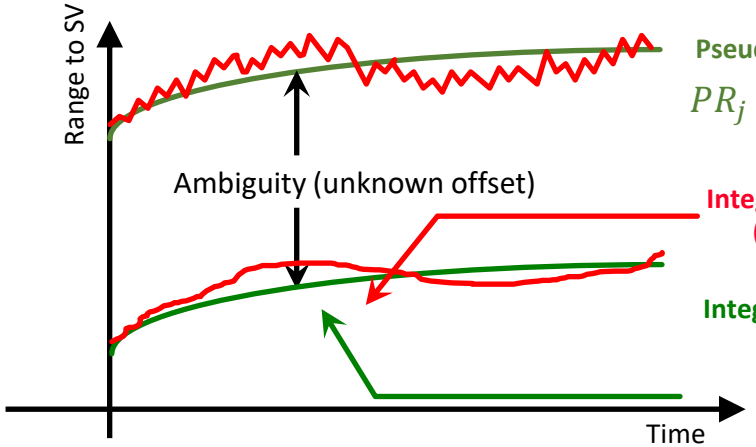
“Raw” Measurements



$$PR_j = R_j + c\delta t_u + \delta R_{PR}$$

Pseudorange (measured)

Pseudorange (true)

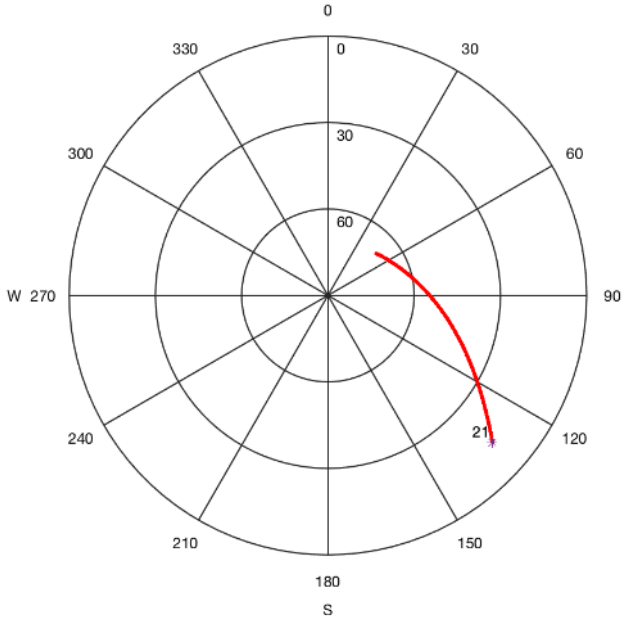
$$PR_j = R_j + c\delta t_u$$

Integrated Doppler (measured)

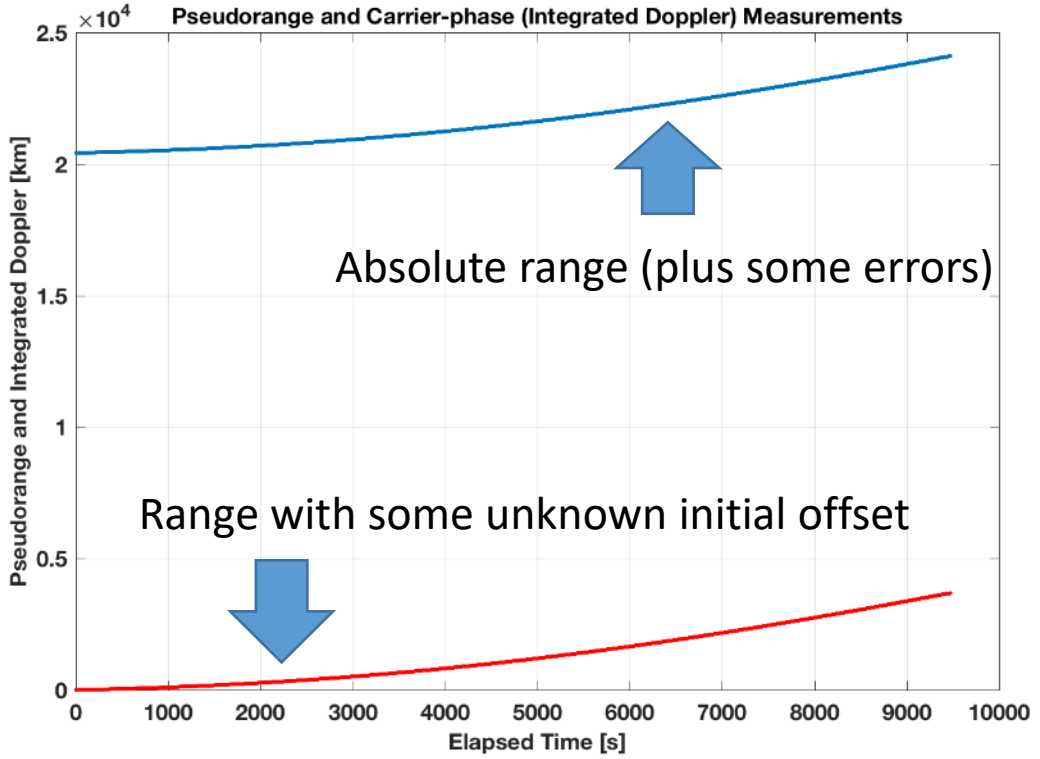
$$\phi_j = R_j + \boldsymbol{N_j\lambda} + c\delta t_u + \delta R_{ID}$$

Integrated Doppler (true)

$$\phi_j = R_j + \boldsymbol{N_j\lambda} + c\delta t_u$$



Satellite starts at an elevation of about 70 degrees and drops to an elevation of about 10 degrees



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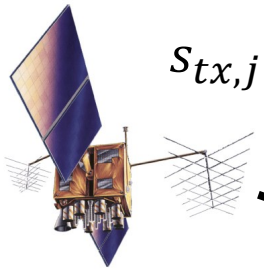
Received Signal – Doppler Frequency

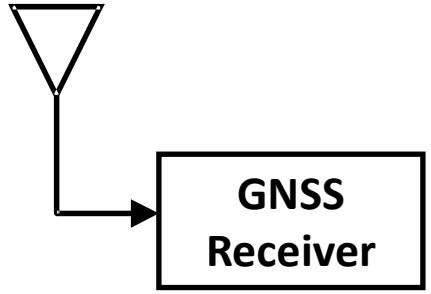


$$s_{tx,j}(t) = A_{tx,j} G_j(t) D_j(t) \sin(2\pi f_{tx} t + \phi_j)$$

line-of-sight dynamics causes a Doppler frequency that is a function of the carrier-frequency and the line-of-site rate

$$\Delta f = -\frac{\mathbf{v} \cdot \mathbf{e}_j}{c} f_{tx}$$

$$s_{rx,j}(t) = \alpha_j A_{tx,j} G_j(t) D_j(t) \sin(2\pi [f_{tx} + \Delta f] t + \phi_j)$$



$\mathbf{v}$: relative 3D velocity of the GNSS receiver (user) with respect to the satellite
c: speed of light

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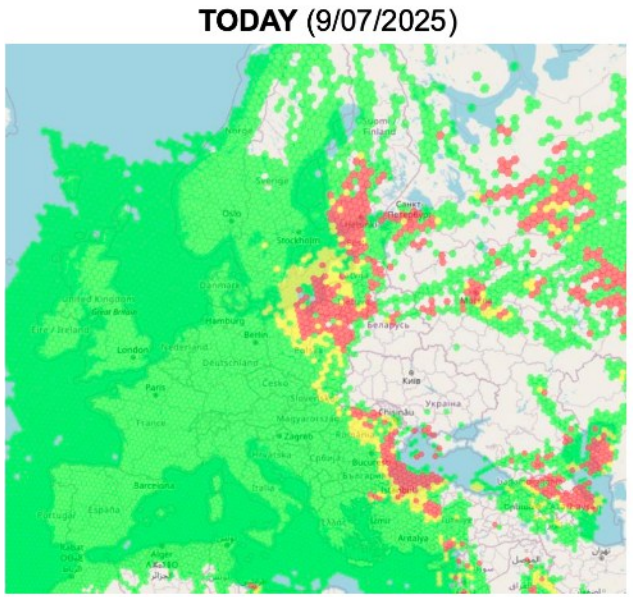
Pseudorange and Carrier-Phase Errors

- Major error sources
  - Propagation delays (ionosphere, troposphere)
  - Satellite orbit and clock errors
  - Receiver measurement errors (noise + dynamics)
  - Multipath (diffuse and specular)
- Others:
  - Interference/jamming
  - Spoofing
  - None-Line-of-sight



Source: Gpsjam.org

Graphical Interpretation of Range Error

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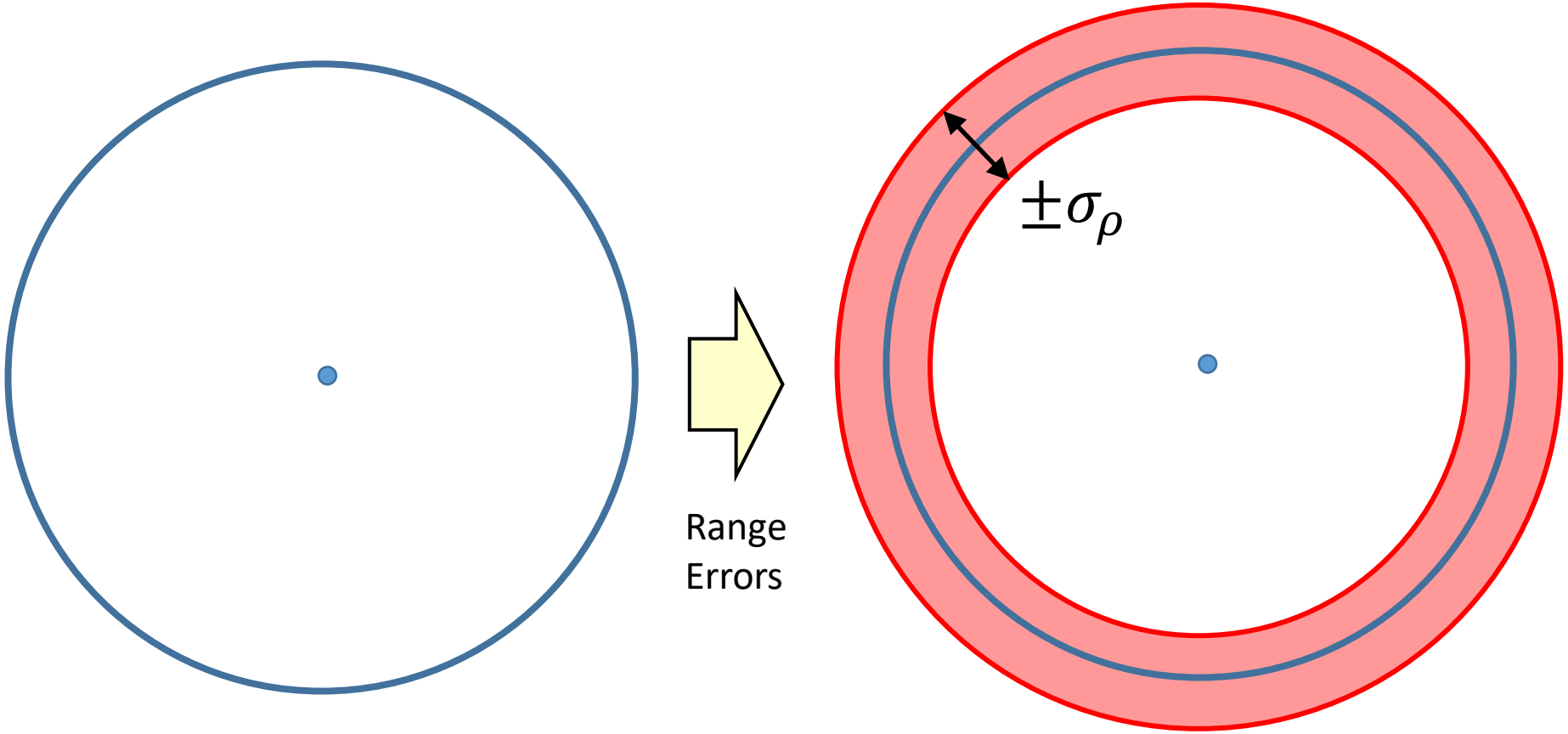

Graphical Interpretation: Geometry

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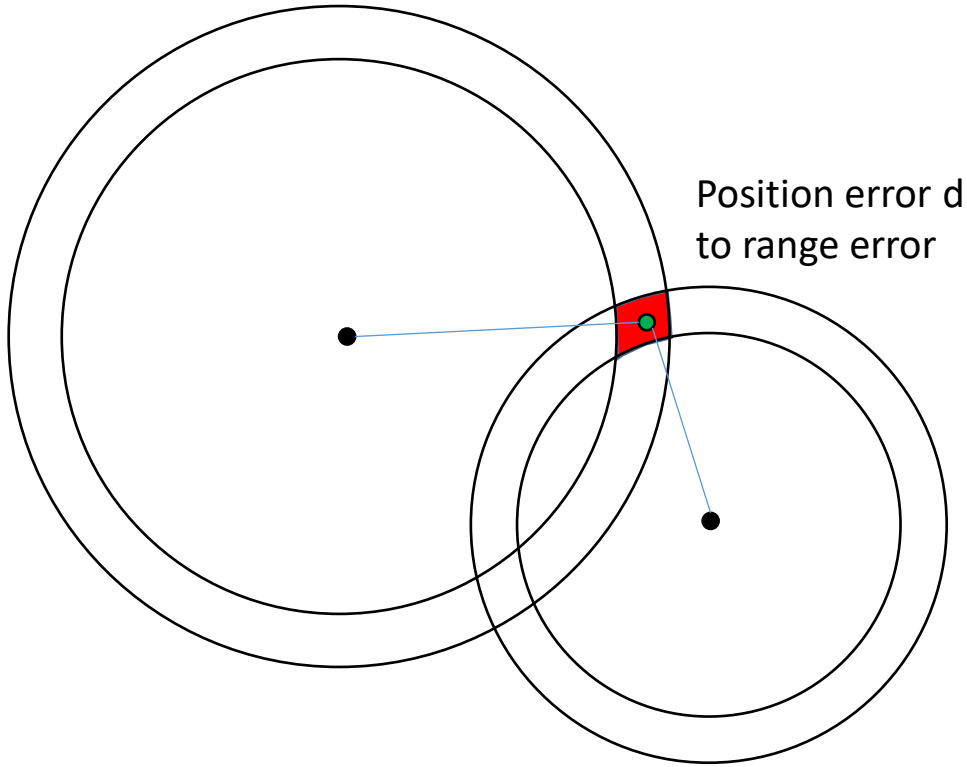
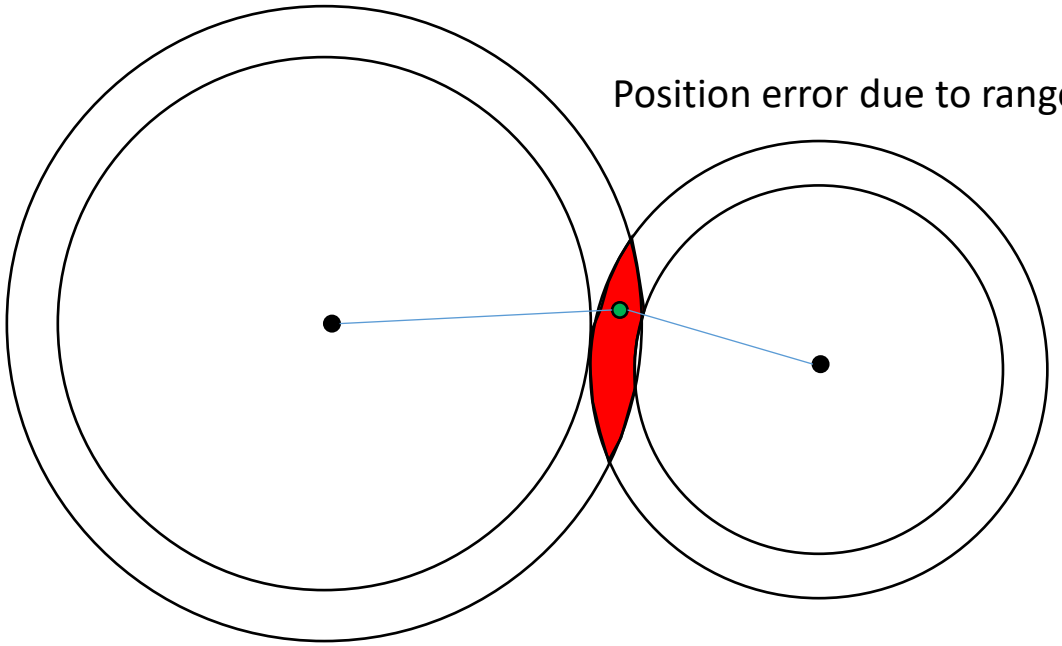
Good geometry



Bad geometry



Larger position error due to worse geometry


Questions?

GNSS Signal Structure – GPS L1 Example

Carrier-frequency
1575.42 MHz

$$s_{tx,G,L1,j}(t) = A_{tx,G,L1,j} G_j(t) D(t) \sin(2\pi f_{L1} t + \alpha_j)$$

Transmitted signal amplitude

Navigation Message data at 50 bps

Carrier signal

SV Carrier-Phase Offset

represents the C/A code (+1 or –1), a pseudo-random noise (PRN) code transmitted at a rate of 1.023 Mcps (chips per second) and repeated every 1 ms (i.e., 1023 chips)

Measurement Equation

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Difference between user clock and satellite clock (here in meters)

GNSS Satellite position

$$\mathbf{z} = \begin{bmatrix} PR_1 \\ PR_2 \\ \vdots \\ PR_N \end{bmatrix} = \mathbf{h}(\mathbf{x}) = \begin{bmatrix} \sqrt{(x_u - x_1)^2 + (y_u - y_1)^2 + (z_u - z_1)^2} + c\delta t_u \\ \sqrt{(x_u - x_2)^2 + (y_u - y_2)^2 + (z_u - z_2)^2} + c\delta t_u \\ \vdots \\ \sqrt{(x_u - x_N)^2 + (y_u - y_N)^2 + (z_u - z_N)^2} + c\delta t_u \end{bmatrix}$$

$$\Rightarrow \mathbf{z} = \begin{bmatrix} \|\mathbf{r}_u - \mathbf{r}_1\| + c\delta t_u \\ \|\mathbf{r}_u - \mathbf{r}_2\| + c\delta t_u \\ \vdots \\ \|\mathbf{r}_u - \mathbf{r}_N\| + c\delta t_u \end{bmatrix}$$

Use norm to express the distance

$\mathbf{r}_u$: user position
$\mathbf{r}_i$: satellite 'i' position

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PR and CP Error Equations

Pseudorange (PR):

$$PR_j = R_j + c\delta t_u + \delta R_{iono} + \delta R_{tropo} + \delta R_{PR,noise} + \delta R_{PR,mp} + \delta R_{PR,hw} - c\delta t_{SV,j}$$

- R_j: True geometric range
- $c\delta t_u$: User clock error
- δR_{iono}: Error due to delay in Ionosphere
- δR_{tropo}: Error due to delay in Troposphere
- $\delta R_{PR,noise}$: Error due to Thermal Noise
- $\delta R_{PR,mp}$: Error due to Multipath (Reflections)
- $\delta R_{PR,hw}$: Error due to Hardware Delays in Receiver
- $c\delta t_{SV,j}$: Satellite clock and orbit error

Carrier-Phase (CP):

$$\phi_j = R_j + N_j\lambda + c\delta t_u - \delta R_{iono} + \delta R_{tropo} + \delta R_{ID,noise} + \delta R_{ID,mp} + \delta R_{ID,hw} - c\delta t_{SV,j}$$

- $N_j\lambda$: Unknown offset
- $-\delta R_{iono}$: Opposite sign from PR
- $\delta R_{ID,noise}$: Much smaller noise (mm-level)
- $\delta R_{ID,mp}$: Smaller multipath (cm-level)